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“Learning in Speculative Bubbles: An Experiment”

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Abstract

Does traders' experience reduce their propensity to participate in speculate bubbles? This paper studies this issue from a theoretical and an experimental viewpoint. We focus on a game in which bubbles, if they arise, are irrational, as in the Smith, Suchanek, and Williams (1988)'s set up. Our theoretical results are based on Camerer and Ho (1999)'s Experience-Weighted Attraction learning model. Adaptive traders are assumed to adjust their behavior according to actions' past performance. In the long run, learning induces the market to converge to the unique no bubble equilibrium. However, learning initially increases traders' propensity to speculate. In the short run, more experienced traders thus create more bubbles. An experiment shows that bubbles are very pervasive despite the fact that subjects have become experienced. Our estimation of the EWA model also indicates that learning is at work.

Keywords: financial markets, adaptive learning, speculation, bubbles

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1 Introduction

Do traders learn to avoid participating in speculative bubbles? This question is the object of a long-standing debate in the financial economics literature. On the one hand, Smith et al. (1988) propose an experimental design to study speculative bubbles and show that bubbles are less likely but do not disappear with experience. This result is confirmed by King, Smith, Williams, and VanBoening (1993). Dufwenberg, Lindqvist, and Moore (2005) further show that bubbles also diminish with experience when only part of the traders are experienced. On the other hand, more recent papers suggest that bubbles can be rekindled when experienced traders are confronted with new market parameters (Hussam, Porter, and Smith (2008)) and when new traders enter the market in an overlapping-generation experiment (Deck, Porter, and Smith (2014)). The objective of this paper is to study learning in speculative bubbles and to identify, both theoretically and experimentally, the conditions under which experience can be expected to shut down speculation. To achieve this objective, we focus on the bubble game designed by Moinas and Pouget (2013). In this game, trading proceeds sequentially, traders' position in the sequence is random, and prices increase exponentially. When there is a price cap, there is no bubble at the dominance-solvable Bayesian Nash equilibrium: confronted with the highest potential price, a rational trader refuses to buy. Anticipating this behavior, a rational trader receiving the second highest price should also refuse to buy. Proceeding backward, this logic rules out the formation of bubbles (the higher the price cap, the higher the number of iterated reasoning steps needed to reach equilibrium). However, when traders suffer from bounded rationality (in the form of limited depth of reasoning as in cognitive hierarchy models or in the form of random mistakes as in the quantal response models), bubbles can emerge

The bubble game is useful to study learning to speculate for three reasons. First, there is a finite number of decisions individuals can make. This makes it possible to analyze behavior in the bubble game thanks to learning models without making strong auxiliary assumptions.

Second, the bubble game enables to study different types of speculative behavior: speculation that is due to costly mistakes and speculation that arises when traders try to profitably trade on others' mistakes. Third, the bubble game is short enough so that it can be repeated a large number of times. This enables to provide subjects with numerous learning opportunities.

We run an experiment based on the bubble game in a stranger design with a small or with a large number of steps of reasoning required to understand that there is no bubble equilibrium. In our experiment, 132 subjects participate in a bubble experiment similar to Moinas and Pouget (2013) with two treatment variables: the cap on the first price is either 1 or 10,000 (corresponding to a maximum number of steps of reasoning of 2 and 6, respectively) and the number of replications of the experiment is either 10 or 20.

Our experimental findings are as follows. When the cap is 1, there are few steps of reasoning and we find that the probability to participate in bubbles tends to decrease over time. When the cap is 10,000, this probability tends to increase. Thus, in our experiment, we observe bubbles that do not vanish with experience. We run panel logit regressions to study individual speculative behavior. These regressions show that subjects' behavior changes depending on the outcomes of past actions. This is an evidence of adaptive learning. Our experimental results, at the market and at the individual levels, suggest that individual learning does not immediately convert into markets experiencing less bubbles.

From a theoretical standpoint, we capture traders' learning process using Camerer and Ho (1999)'s Experience-Weighted Attraction model. This adaptive learning model is general in the sense that it nests belief-based learning and reinforcement learning. Traders' choices depend on the various actions' attractions, i.e., accumulated past payoffs. A crucial parameter in this model is the imagination parameter. When it is equal to 0, agents only reinforce chosen actions, as it is the case in reinforcement learning. When the imagination parameter is greater than 0, agents also reinforce actions that were not actually chosen, as it is implicitly assumed in belief-based learning. Traders' attractions are transformed into choice probabilities via a logistic function with a given payoff responsiveness parameter. When this

responsiveness parameter is 0, players choose each action with the same probability, while when it is infinite, players choose with probability one the action with the highest attraction. Camerer and Ho (1999)’s model is useful because it enables us to study whether adaptive traders’ speculating behavior depends on the type of learning process.

We first provide analytical results. In the long term, the market converges to the unique dominance-solvable no-bubble equilibrium. This is in line with the analysis of Milgrom and Roberts (1991). However, in the short term, the propensity to speculate can increase. In order to study speculation in the intermediate term, we run simulations of the bubble game with 1,000 independent trials that each includes 1,000 successive runs. We show that learning initially increases traders’ propensity to speculate. More experienced traders thus initially create more bubbles. The effect of imagination on speculation depends on whether traders can observe the choice of the next trader in the market sequence even if they do not speculate.

To better understand the type of learning at work in our data, we structurally estimate the learning model on our experimental data. We find an imagination parameter of 0.45, not statistically different from zero. This indicates that imagination is not an important ingredient for learning in our speculation setting. Our estimate of the responsiveness parameter is 0.2. This suggests that there is a lot of noise in individual behavior. Vuong tests indicate that the learning model fits the data better than a no-learning model in which agents’ choice probabilities are constant and similar to the first period of play. Our structural estimations of the learning model thus suggests that learning is indeed at work in our experiment. Overall, our results show that learning does not easily shut down speculative bubbles.

The rest of the paper is organized as follows. The next section offers a review of the relevant literature. Section 3 presents the bubble game and the experimental setup. Section 4 presents the experimental results. Section 5 offers a theoretical analysis of bubbles based on adaptive learning and estimates the EWA model. Section 6 concludes.

2 Related literature

In their seminal contribution, Smith et al. (1988) propose an experimental setup in which (irrational) bubbles can be studied, and they show that such bubbles arise in their experiments. This contribution has triggered an extremely large number of experimental and theoretical work on bubble formation (see for example, Deck et al. (2014) for a discussion of a number of the follow-up studies to Smith et al. (1988), and Caginalp and Ermentrout (1990) for a mathematical model of speculation inspired by Smith et al. (1988)).

Smith et al. (1988) also show that several replications with the same subjects were requested to attenuate the emergence of bubbles. Their results were replicated in numerous subsequent papers including King et al. (1993) who show that bubbles emerge in a variety of market environments and also conclude that three replications of the experiments were necessary to shut down irrational speculation. Dufwenberg et al. (2005) further shows that bubbles diminish even when part of the traders are experienced.

The impact of experience on bubble formation is however not completely clear since more recent papers suggest that bubbles can be rekindled when experienced traders are confronted with new market parameters (Hussam et al. (2008)) and when new traders enter the market in an overlapping-generation experiment (Deck et al. (2014)). Our paper speaks to this debate and studies, both theoretically and experimentally, the conditions under which experience can be expected to shut down irrational speculative bubbles.

Duffy and Ünver (2006) use the Smith et al. (1988) bubble environment in order to study the behavior of artificially-intelligent agents. They show that near-zero intelligence traders generate irrational bubbles and crashes when traders are endowed with some foresight ability and adopt a price setting behavior with anchoring effects and within exogenously defined bounds. We complement the analysis of Duffy and Ünver (2006) by studying how learning affects speculative behavior.

Some papers have studied learning to speculate in an environment where it is rational. In

this case, the surprising result is that speculation did not occur as much as could be expected. Duffy and Ochs (1999) for example show that subjects had difficulties coordinating on the speculative equilibrium in a money experiment. They show that this lack of speculation was related to a tendency of subjects to rely on past choices' payoffs rather than on the profit to be expected from speculation. Duffy (2001) extends the analysis of Duffy and Ochs (1999) and shows that the propensity to speculate increases when profit opportunities are more frequent or less risky in line with an agent-based model that incorporates some features of adaptive learning. We complement these articles by proposing a simple irrational bubble experiment in which a more general adaptive learning model can be tested and estimated.

Learning has also been extensively studied both by experimentalists and by empiricists. For example, Nagel and Tang (1998) propose an experiment to study learning in the normal-form centipede game. Their results suggest that learning is not necessarily conducive to equilibrium. Duffy and Nagel (1997) show that directional learning in a beauty contest game can induce some agents to adjust their play away from equilibrium as they get experienced. This phenomenon can also be present in the adaptive learning model that we study and we show that it plays an important role in the fact that experience might induce more bubbles in the short-run. The result that more experienced traders may speculate more echoes the findings of De Martino, O'Doherty, Ray, Bossaerts, and Camerer (2013) indicating that people who are more sophisticated, in the sense that they are better able to infer intentions from others' actions, are more prone to speculate in bubbles.

Empiricists have also found clever ways at identifying learning and its impact on various financial and macroeconomic variables. For example, Kaustia and Knüpfer (2008) find that the outcomes of IPOs in which they have participated affect future investment behavior of individuals. Malmendier and Nagel (2011, 2016) show that investors' expectations regarding stock returns and inflation depend on their own economic experience rather than on the entire sample of information available. This indicates that learning based on experience is at work in the field. We complement these studies by showing how adaptive learning can

fuel speculative bubbles. In particular, our theoretical analysis based on adaptive learning complements the analyses of Caginalp and Ermentrout (1990), Caginalp and DeSantis (2011), and Barberis, Greenwood, Jin, and Shleifer (2015) which show that extrapolative behavior can trigger bubble formation.

3 The bubble game and the experimental setup

3.1 The bubble game

To study how experience affects speculative behavior, we focus on the bubble game designed by Moinas and Pouget (2013). This game features a sequential market for a valueless asset. There are three traders. Each of them is randomly assigned to a position in the market sequence: a trader can be first, second, or third in the sequence with probability $\frac{1}{3}$. Traders do not have information about their positions but can infer some information from the price at which they are offered to buy the asset.

For simplicity, prices are assumed exogenous. The first trader is offered a price 10^n , where n is random and follows a geometric distribution of parameter $\frac{1}{2}$: $P(n = k) = \frac{1}{2}^{k+1}$, where $k \in \{0, 1, 2, 3, \dots\}$. Each subsequent trader is (potentially) offered a price that is ten times higher than the previous price. This setting is such that no trader can ever be sure to be last in the market sequence despite prices revealing some information regarding traders' position. In this case, Moinas and Pouget (2013) show that rational bubbles can arise at the Nash equilibrium. However, they show that, when there is a price cap, no bubble can arise at equilibrium if rationality is common knowledge: the unique dominance solvable equilibrium involve all traders refusing to buy the asset.

We consider a repeated version of the above bubble game with a cap on the first price. At the beginning of each period, each trader is endowed with 1 monetary unit that he can use to buy the asset. If a trader in the previous position, if any, decided not to buy, a trader obtains a trading profit of 0 since the game has stopped before his decision could matter.

Being proposed to buy at a price P , if a trader chooses not to buy the asset, his profit from this action is 0. If a trader decides to buy the asset, we assume that an outside financier provides the remaining funds $P - 1$ and shares the proceeds proportionately. The trader thus obtains a trading profit of 9 if he is able to sell the asset to the next trader, and a trading loss of -1 if he cannot sell back.

3.2 Experimental design

To test whether subjects' experience influences bubble formation, we run an experiment in which subjects participate in a sequence of independent bubble games. In order to study the influence of complexity on learning and speculation, we vary the maximum number of steps of reasoning required to reach the Nash equilibrium. The maximum number of steps of reasoning is 2 when the cap on the first price is 1 while this number is 6 when the cap is 10,000. The number of replications is either 10 or 20. We thus have four treatments: 30 subjects have participated in an experiment with a cap at 1 and 10 replications, 72 subjects with a cap at 10,000 and 10 replications, 12 subjects with a cap at 1 and 20 replications, and 18 subjects with a cap at 10,000 and 20 replications. A total of 132 subjects have participated in our experiment. Subjects are junior and senior undergraduates in business administration at the University of Toulouse.

In the experiment, we use a quasi-strategy method: the price proposed to the first subject in the trading sequence is randomly drawn and the prices that could be proposed to the next two subjects are functions of the first price. Subjects, according to their position in the sequence, are proposed the corresponding price and indicate whether, if they were proposed this price, they would buy or not the asset. This enables us to observe all three traders' decisions, even if the first trader decides not to buy (a case in which the bubble does not start and in which we could not have observed what other traders would have done). Because we do not observe subjects' decisions at all potential prices but only at the price that they could be proposed given the draw of the first price, we are not in a full fledged strategy method.

Subjects are offered one unit of experimental currency at the beginning of each replication. Their trading payoffs are determined as indicated in Section 3.1. If they are not offered to buy or decide not to buy the asset, their trading profit is zero. If they are able to buy and resell the asset, their trading profit is 9. If they are not able to resell the asset to the next subject, they make a trading loss of -1 . The exchange rate we use in the experiment is one Euro per experimental monetary unit. The instructions of the experiment with a cap at 10,000 are in Appendix A. The average gain in the experiment was 30 Euros. The overall minimum and maximum payments in the experiment, including a 5 Euros show-up fee, were 5 Euros and 134 Euros. The experiment lasted between one hour and one hour and a half.

4 Experimental results

Our findings are summarized in Figure 1 which displays the evolution of average probability to buy across replications (the solid red line on the graph). When the cap on the first price is 1 (top graph), there are few steps of reasoning to reach the equilibrium and behavior seems to converge to the no bubble equilibrium: the average probability to buy shows a decreasing trend. However, when the cap on the first price is 10,000 (bottom graph), there is no monotonically negative trend in the probability to buy.

To further document the evolution of speculative behavior in the experiment, Figure 2 displays the probability to buy across replications for the different price levels. The different price levels are associated with different number of steps of reasoning and with a different conditional probability to be last. When the cap on the first price is 1 (top graph), the probability to buy seem to decrease at all price levels even if not monotonically. This is consistent with the experimental findings of King et al. (1993) and Dufwenberg et al. (2005).

When the cap on the first price is 10,000 (bottom graph), behaviors seem different across prices. When the price is 1, the probability to buy is constant at 100% after the first period. When the price is 10, 100 and 1,000, the probability to buy seems to increase with experience.

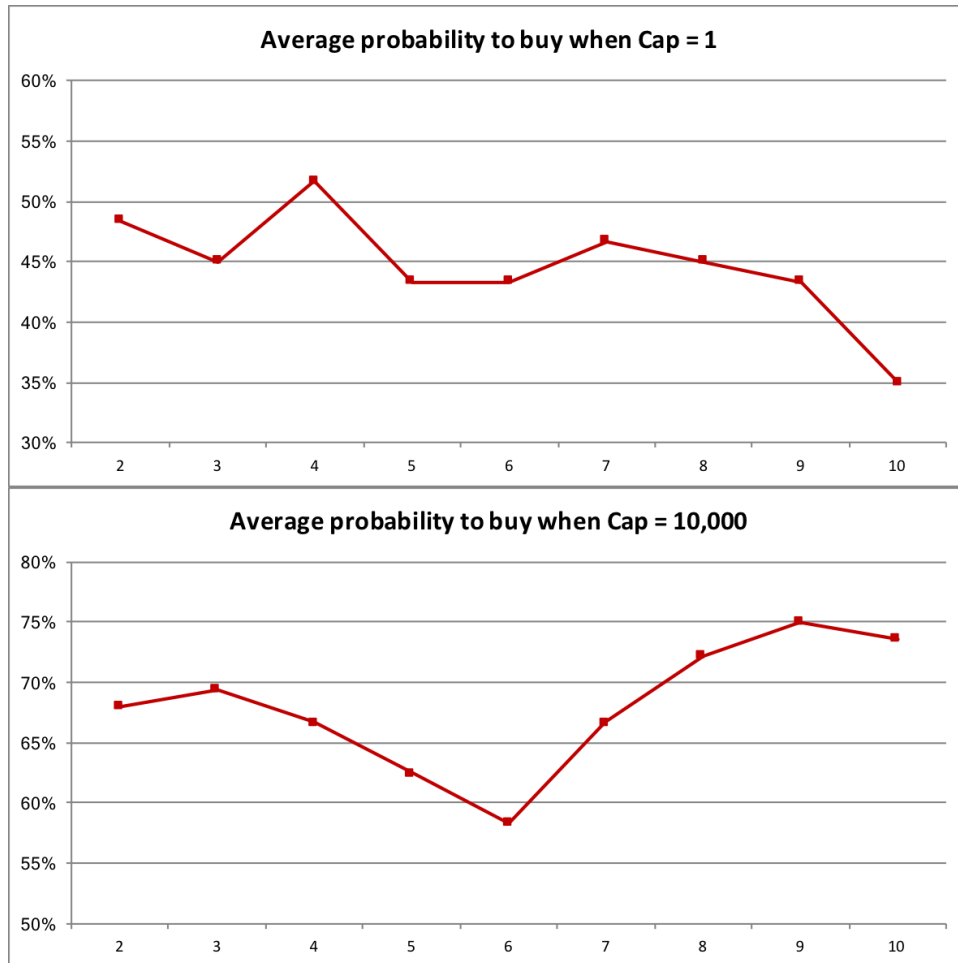


Figure 1: Data and predictions from learning models.
Probability to buy by period averaged across subjects and prices, as measured in the data.

For the remaining prices, the probability to buy trends upward, downward or is flat. Overall, there is some evidence that behavior evolves with experience but not always towards less speculation.

In order to study whether we can identify learning in our data, we set up a panel logit regression. We regress the likelihood that a subject buys the overvalued asset onto various explanatory variables that reflect the probability to be last in the market sequence, the number of steps of reasoning, and the result of past actions. For example, $\mathbb{1}_{Step=1 \text{ or } 2}$ is a dummy variable that takes the value 1 if the number of steps of reasoning is 1 or 2, $\mathbb{1}_{0 < P(last) < 1}$ takes the value 1 if the probability to be last is strictly between 0 and 1, and $\mathbb{1}_{\text{bought and lost at least once}}$ indicates that, in at least one previous replication, the subject has bought the asset and could not sell it back. We control for subject and replication fixed effects. In the analysis, 4 subjects were dropped because they never bought and 5 because they always bought.

The results of the panel logit regression are in Table 1. The first regression specification of Table 1 replicates the analysis of the one-shot game in Moinas and Pouget (2013) on subjects' decisions in the first period. We find that, as in Moinas and Pouget (2013), when the probability to be last decreases and when the number of steps of reasoning increases, subjects are more likely to speculate and buy the overvalued asset in the hope to resell it at a profit. We complement these results, in the next two specifications, by showing how past outcomes influence speculation, when subjects play multiple periods. We find that the propensity to speculate goes down after a subject has bought the asset but was unable to sell it back (the propensity to speculate appears larger after a subject has experienced a profit from speculating in the past, but this result is not statistically significant). The third specification indicates that this effect is significant especially when subjects are sure not to be last. These results are consistent with adaptive learning being at work in our experimental bubble market. The fact that they have already been proposed the highest potential price seems not to affect the propensity to speculate, indicating that their speculation is not driven

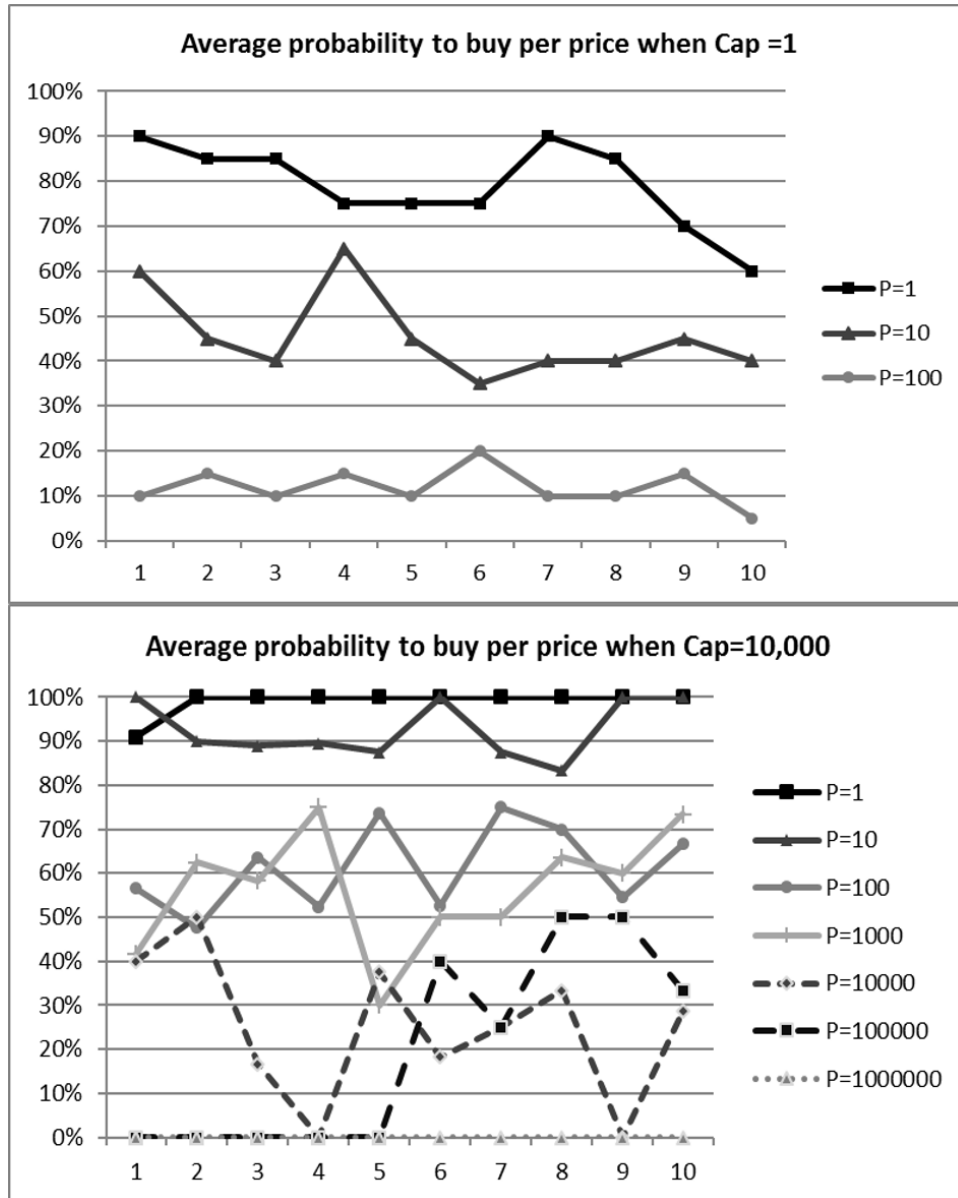


Figure 2: Data on the probability to speculate per price.
Probability to buy per price by period averaged across subjects, as measured in the data.

by their lack of understanding of the game structure.

To summarize our results, Figure 3 plots the empirical frequencies of bubbles in our experiment. Large, medium and small bubbles correspond to situations in which all three subjects in a market have decided to buy the asset, only the first two subjects have bought, and only the first subject has bought, respectively. The upper left graph displays the results for the case in which there is a cap on the first price at 1 and there are 10 replications of the game. It shows that the market experiences less and less bubbles over time. During the first replication, no bubble occurs 30% of the time while, during the 10th replication, no bubble occurs 80% of the time. As shown in the bottom left graph, bubbles are not eliminated by experience when there is a large number of steps of reasoning as is the case when the cap on the first price is 10,000. The two graphs on the right of the Figure show that the likelihood of large bubbles is very high even after 20 replications of the experiment. More precisely, as shown in the bottom right graph, large bubbles occurs around 30% of the time in the first replication but almost 70% of the time in the twentieth replication when the cap on the first price is 10,000. Overall, our experimental results suggest that learning is not likely to shut down speculative bubbles quickly, especially when a high number of steps of reasoning is needed.

5 A theory of bubbles based on adaptive learning

We present a model based on adaptive learning in which traders' behavior evolves over time. We then estimate the model using our experimental data.

5.1 The adaptive learning model

In our model, traders are assumed to adopt an adaptive behavior and adjust their choices according to past performance. We capture adaptive behavior using a simplified version of Camerer and Ho (1999)' Experience-Weighted Attraction model that nests reinforcement

| | (1) | (2) | (3) |
|--|---------------------|----------------------|----------------------|
| | Buy Decision | Buy Decision | Buy Decision |
| Constant | -2.251** (-3.03) | -6.456*** (-5.49) | -6.819*** (-5.58) |
| $\mathbb{1}_{Step=1 \text{ or } 2} \times \mathbb{1}_{0 < P(last) < 1}$ | 1.558 (1.37) | 15.10 (0.02) | 14.72 (0.02) |
| $\mathbb{1}_{Step=1 \text{ or } 2} \times \mathbb{1}_{P(last)=0}$ | 3.350*** (4.04) | 5.235*** (9.05) | 5.681*** (9.03) |
| $\mathbb{1}_{Step \geq 3} \times \mathbb{1}_{0 < P(last) < 1}$ | 2.308** (2.83) | 18.03 (0.02) | 17.65 (0.02) |
| $\mathbb{1}_{Step \geq 3} \times \mathbb{1}_{P(last)=0}$ | 5.619*** (4.46) | 21.66 (0.03) | 21.97 (0.03) |
| Accumulated gains | | 0.0116 (1.31) | 0.0113 (1.26) |
| $\mathbb{1}_{\text{has observed the max price at least once}} \times \mathbb{1}_{P(last) < 1}$ | | -0.349 (-0.78) | |
| $\mathbb{1}_{\text{bought and lost at least once}} \times \mathbb{1}_{P(last) < 1}$ | | -1.255*** (-3.68) | |
| $\mathbb{1}_{\text{bought and won at least once}} \times \mathbb{1}_{P(last) < 1}$ | | 0.441 (1.28) | |
| $\mathbb{1}_{\text{bought and lost at least once}} \times \mathbb{1}_{0 < P(last) < 1}$ | | | -0.766* (1.79) |
| $\mathbb{1}_{\text{bought and lost at least once}} \times \mathbb{1}_{P(last)=0}$ | | | -1.678*** (-3.93) |
| $\mathbb{1}_{\text{bought and won at least once}} \times \mathbb{1}_{0 < P(last) < 1}$ | | | 0.484 (1.01) |
| $\mathbb{1}_{\text{bought and won at least once}} \times \mathbb{1}_{P(last)=0}$ | | | 0.251 (0.60) |
| Observations | 132 | 1230 | 1230 |
| Time and Subject Fixed Effects | No | Yes | Yes |

Table 1: Regressions of the probability to speculate.

Regression of the likelihood that a subject buys the overvalued asset onto various explanatory variables that reflect the probability to be last in the market sequence, the number of steps of reasoning, and the result of past actions. We control for subject and replication fixed effects. In the panel analysis (columns 2 and 3), 4 subjects were dropped because they never bought and 5 because they always bought. T-statistics are in parentheses. *, **, and *** denote statistical significance (two tailed) at the 10%, 5%, and 1% levels, respectively.

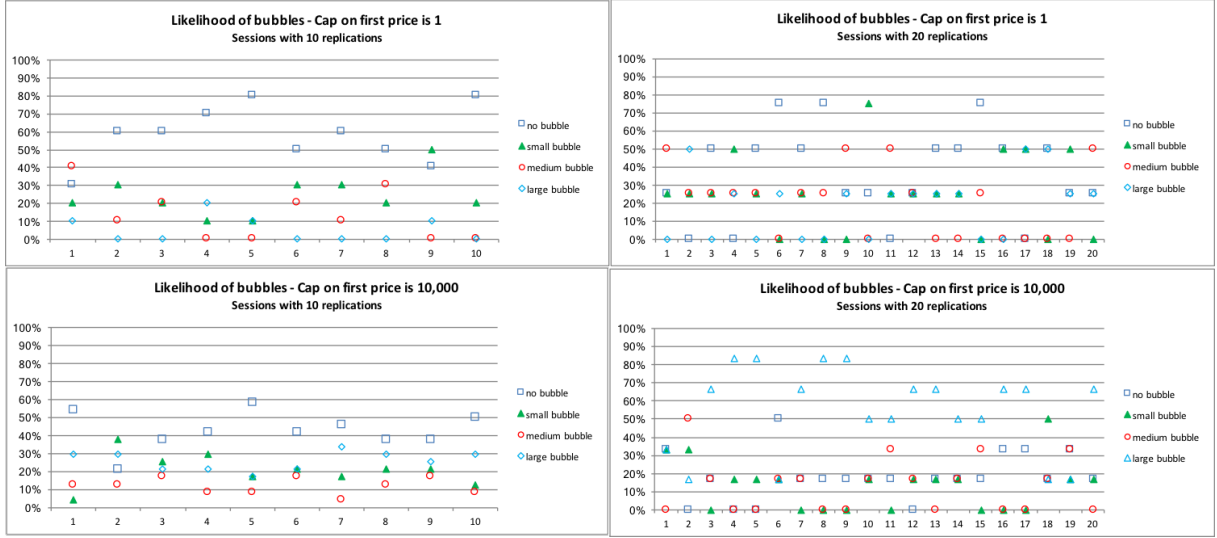


Figure 3: Data on the frequency of bubbles.

Large, medium and small bubbles correspond to situations in which all three subjects in a market have decided to buy the asset, only the first two subjects have bought, and only the first subject has bought, respectively.

and belief-based learning. Let $a_i(t)$ denote the action chosen by agent i at date t . An action is denoted by j with $j \in \{B, \emptyset\}$ (B stands for a decision to buy, \emptyset for a decision not to buy). In our repeated trading game, at a given price P , the attraction of action j for trader i at period $t + 1$ is updated as follows:

$$A_i^j(t+1|P) = \begin{cases} A_i^j(t|P) + [\delta + (1 - \delta)1_{j=a_i(t)}] \pi(j, a_{-i}(t)) & \text{if } i \text{ is proposed price } P \\ A_i^j(t|P) & \text{otherwise} \end{cases} \quad (1)$$

$\pi(j, a_{-i}(t))$ is the profit received by trader i if he chooses action j given other traders choosing action $a_{-i}(t)$. $1_{j=a_i(t)}$ is an indicator function, which is equal to 1 if $j = a_i(t)$ and 0 otherwise. In equation (1), the first part represents how the attraction of trader i 's action j conditional on price P is updated if trader i is proposed a price P at period t ; the second part represents how this attraction is updated if trader i is not proposed price P at period t .¹

¹Note that traders may not be proposed any price in a given period, when one of the previous traders has decided not to buy.

δ is the imagination parameter that controls how much agents are able to display counterfactual reasoning. When $\delta = 0$, trader i reinforces the profit of action j only if it is selected at period t , while when $\delta > 0$, trader i reinforces the profit of action j no matter whether it is actually chosen or not, provided that he is proposed the price P .

This adaptive learning model captures both the law of actual effect and the law of simulated effect. The law of actual effect means that the attraction of an action is adjusted only if this action has been selected ($\delta = 0$). If the action generates a positive profit, this action will be more attractive, otherwise, it will be less attractive. This law is at the core of reinforcement learning (see, for example, Roth and Erev (1995)). The law of simulated effect indicates that the attraction of an action is adjusted according to the profit it could have generated even if it has not been selected ($\delta > 0$). This law is at the core of belief-based learning (see, for example, Fudenberg and Levine (1998)).

According to the attraction, trader i decides the probability to choose action j at period $t + 1$ as follows:

$$Pr_i^j(t + 1|P) = \frac{1}{1 + e^{\lambda[A_i^{-j}(t|P) - A_i^j(t|P)]}} \quad (2)$$

where λ represents the responsiveness of traders to attractions. Equation (2) indicates that the probability for trader i to choose action j is determined by its relative attraction in the previous period.

To close the system, the initial values of $A_i^j(0|P)$ need to be specified. We set $A_i^j(0|P) = 0$, i.e. traders initially choose each action with the same probability.²

5.2 Simulation results

Our simulations consider two cases as in the experiments: price cap is 1 and 10,000. For both cases, we have a 2×2 design with δ being equal to 0 or 1, and λ being equal to 1 or

²Our version of the EWA learning model corresponds to the original version of Camerer and Ho (1999) with $\rho = 0$ and $\phi = 1$.

1,000. Our simulation proceeds as follows: for a given set of parameters, each simulation contains 1,000 independent trials; each trial contains 1,000 runs, where each run represents one trading session. In our experiments, at the end of each period, subjects have been provided with information regarding the action of the next subject in the market sequence only when he has bought (and when the previous trader, if any, has also bought). Therefore, imagination, measured by δ , only matters in the case where a trader receives the highest possible price and decides not to buy. In this case, the trader can imagine he would have lost -1 if he had bought the asset.³

5.2.1 Individual trading behavior: Cap on the first price is 1

Figure 4 depicts the simulated average probability to buy at different prices. We first look at the case in which $\delta = 0$ and $\lambda = 1$ (upper left graph). Since the last trader can never sell back the asset, he cannot gain if he chooses to buy and therefore he learns not to participate in trading this asset. Due to lack of experience, the last trader initially chooses to buy with a high probability (50% given the initial attractions). In this case, the second trader can sell the asset at 10 times higher price with a high probability. He can gain from trading, thus initially leading to more speculation. As time goes by, the last trader learns not to speculate and the second trader is then less prone to speculate. The same pattern occurs for the first trader, the only difference being that the first trader is more inclined to buy the asset due to the higher speculation of the second trader. As a result, the convergence to no speculation is very slow for the first trader.

Speculative behavior does not change dramatically if δ increases to 1, i.e., the maximal level of imagination thanks to which agents adapt their behavior in response to the payoffs they could have had. The simulations (bottom left) show that imagination leads to quicker learning: initially, agents who are proposed the maximum price learn quickly not to speculate,

³A trader who buys can always use his imagination to compute the payoff he would received if he does not buy. However, since the profit from not buying is equal to zero, imagination cannot affect the attraction of the action not to buy.

which induces the occurrence of bubbles to decrease more rapidly than in the case without imagination.⁴

An increase in traders' responsiveness to attractions, λ , reduces (but does not eliminate) speculation, especially when $\delta = 0$. Note that even in this case, the probability to buy of the last trader may converge very slowly to zero (as in the upper right graph): when traders early in the market sequence have already learnt not to speculate, the last trader is very rarely offered the opportunity to buy (and thus to learn, because we assume that attractions are specific to a given price and are updated only when this price is proposed to traders). This result might explain why bubbles can be rekindled when experienced traders are confronted with new market parameters as in Hussam, Porter and Smith (2008).

These results on individual behavior shed some light on the aggregate market behavior. The fact that the first (respectively, last) traders in the market sequence initially learn to (respectively, not to) speculate indicates that the likelihood of bubbles initially increases and then takes some time to converge to zero.⁵ Second, imagination induces the first traders in the market sequence to learn more strongly to speculate in the short run but also to learn more quickly to converge to no speculation.

5.2.2 Individual trading behavior: Cap on the first price is 10,000

As shown in Figure 5, raising the cap on the first price significantly fosters traders' speculation: with a price cap at 1, the trading approaches to the no bubble scenario within the 1,000 simulation runs. However, when the price cap is 10,000, traders' propensity to speculate is very high even after 1,000 runs: the probability to buy the asset at prices of 1, 10, 100 and 1,000 ends up greater than 70% when $\lambda = 1$. When $\lambda = 1,000$, the probability to buy decreases with experience but at a slower pace than when the price cap is 1.

⁴In Appendix B, we show that, when traders observe the actions of the subsequent traders and $\delta = 1$, the propensity to speculate is initially stronger than with no imagination because imaginative traders who are proposed low prices realize that they could have obtained a higher payoff by buying the asset if subsequent traders decided to buy.

⁵The fact that some traders see their propensity to speculate initially increase depends on the fact that some traders in the market sequence buy with a probability larger than 10%.

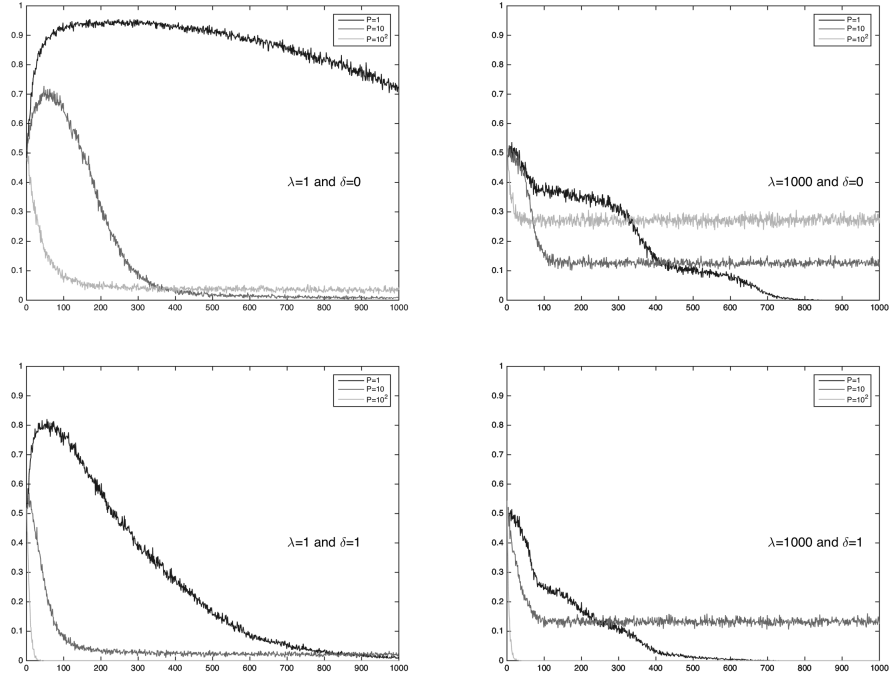


Figure 4: Probability to speculate per price in the EWA learning model when the cap on the first price is 1.

This figure displays the average probability to buy in the bubble game as provided by the EWA learning model of Camerer and Ho (1999). We set $A_i^j(0|P) = 0$.

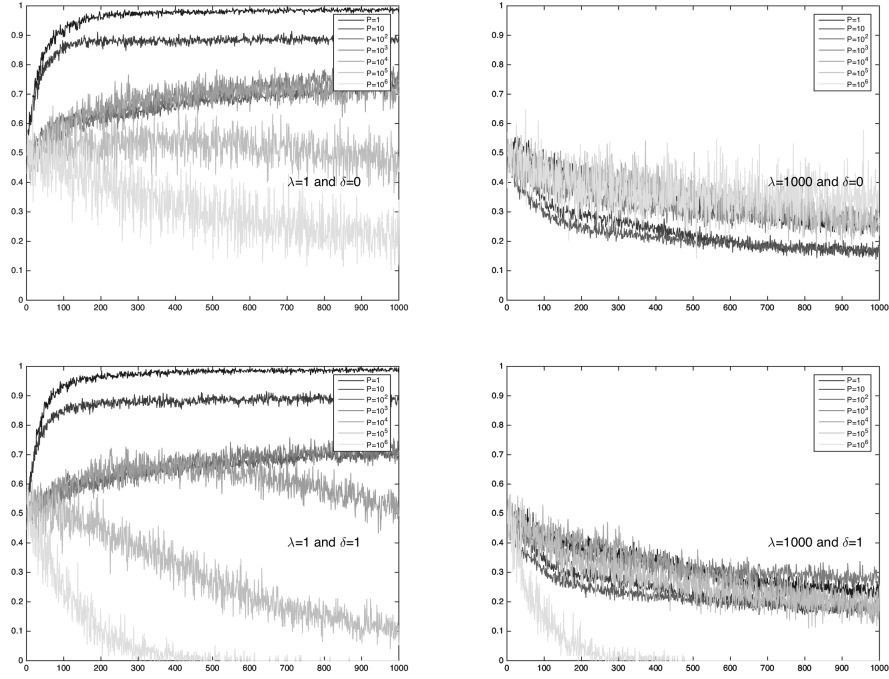


Figure 5: Probability to speculate per price in the EWA learning model when the cap on the first price is 10,000.

This figure displays the average probability to buy in the bubble game as provided by the EWA learning model of Camerer and Ho (1999). We set $A_i^j(0|P) = 0$.

5.3 Analytical results

In this subsection, we complement our simulation results and provide analytical results on the propensity to speculate. We focus on the long run and the short run. The intermediate case is much more difficult to analyze but it can be qualitatively understood by comparing the short and the long run. The proofs are in Appendix C.

Our first proposition focuses on the long run convergence of the adaptive learning model applied to the bubble game. This analysis is an application of the more general case studied in Milgrom and Roberts (1991).

Proposition 1 *Assume that $\lambda > 0$. In the long run, the trading game converges to the no bubble equilibrium.*

This result comes from the fact that the bubble game with a price cap is dominance solvable. The trader with the highest price can only loose money by speculating. Adaptive learning will thus lead to a decrease in the probability to buy the asset at this price towards zero. Once this probability is low enough, the trader with the second highest price will also see his probability to buy converge to zero. Eventually, all traders will see their probability to buy converge to zero.

The short run situation is very different from the long run and is summarized in the following proposition.

Proposition 2 *Assume that $\lambda > 0$. At period 1, traders randomly select between buy or no buy with equal probability. At period 2, if a trader observes a price of 1 or 10, inducing that he cannot be last in the market sequence, his expected probability to buy increases. If a trader observes a price which is strictly higher than 10 and lower than the highest price, his expected probability to buy increases if λ is small and decreases otherwise. If a trader observes the highest price, his expected probability to buy always decreases.*

Overall, we find that in the short run the propensity to speculate can increase especially when the trader is sure he cannot be the last in market sequence. This is in line with our

experimental result as displayed in Table 1. We also find that, in the long run, traders learn not to speculate. In the intermediate periods, we thus expect that a trader’s propensity to speculate at a given price will decrease at some point in time as soon as the probability to speculate of traders at the next price has become low enough.

5.4 Structural estimation of the learning model of speculation

We now estimate our version of the EWA learning model on the data generated in our experiment using the Maximum Likelihood technique. This is useful in order to study the type of adaptive learning that is at work: by estimating the EWA learning model, we can determine whether people tend to use the law of actual effect only (i.e., whether the estimate of the imagination parameter δ is zero) or whether they also use the law of simulated effect (the estimate of δ is larger than zero).

Initial attractions are estimated from the behavior in the first replication of the experiment and in the one-period experiment of Moinas and Pouget (2013).⁶ For the Maximum Likelihood estimation, we restrict the set of parameter values as follows: δ is between 0 and 1 with increments of 0.05, and λ between 0 and 10 with increments of 0.05. We compute the likelihood of the model for all the potential parameters’ values. The estimated parameters correspond to the ones associated with the highest likelihood.

To compute confidence intervals for the parameters, we estimate the model several times after dropping one or more replications at the end of the experiment. We do this in order to keep the time dependency that is inherent to repeated experiments: we re-estimate the model on 6, 7 or 8 first replications only (after the first replication which is not included in the learning parameters’ estimation). Moreover, we also re-estimate the model by dropping one of the experimental sessions. Overall, this provides us with 35 replications. For each parameter, we drop the two minimum parameter values and the two maximum ones to get an 11% confidence interval.

⁶We used data from the one-shot bubble game of Moinas and Pouget (2013) in order to have observations at all prices, which is not the case in the present experiment.

The results are displayed in Table 2. The first column corresponds to the model estimation. Because subjects do not observe the subsequent trader’s decision, the parameter δ can be only be identified for subjects who are proposed the highest price and decide not to buy. The three other columns are offered for comparison. The second column corresponds to the case in which δ is set to 0, i.e., traders learn but have no imagination; the third column corresponds to the case in which δ is set to 1, i.e., traders learn and have imagination. Comparing these two columns enables one to assess the importance of the law of simulated effect in our bubble experiment. Finally, the last column provides the likelihood of a model in which there is no learning and the behavior of agents is fixed and similar to behavior in the first period.

The results show that in our data, the learning model has a higher likelihood than the fixed behavior model. The difference in log-likelihood is statistically significant according to a Vuong test (the p-value is lower than 1%). This is in line with our previous regression results indicating that adaptive learning is at play in our experiments and is an important element in speculative behavior.

We also find that imagination does not play a large role in our experiment. According to Vuong tests, the maximum likelihood of the learning model is significantly larger when δ is unconstrained than when it is constrained to 0 or to 1 (the p-values are equal to 0.03). The improvements however appear relatively small compared to the improvement in maximum likelihood between the learning models and the no-learning benchmark. This is not to say that imagination is not important for speculation. Indeed, in our experiment, when a subject decided not to buy, he was not told what the next subject in the market sequence (if any) wanted to do. He thus had no data on which to apply his potential imagination. To study this issue further and test whether imagination is important for speculation decisions, it would be interesting to run an experiment in which subjects were told what the next subject wanted to do.

| | Unconstrained δ | $\delta = 0$ | $\delta = 1$ | No learning |
|-----------------------------------|------------------------|--------------|--------------|-------------|
| λ | 0.20 | 0.20 | 0.20 | > 0 |
| δ | 0.45 | 0.00 | 1.00 | |
| Max L | -571.17 | -571.33 | -571.36 | -594.73 |
| Contribution of cap = 1 | | | | |
| Max L | -275.92 | -276.08 | -276.11 | -290.80 |
| Contribution of cap = 10,000 | | | | |
| Max L | -295.25 | -295.25 | -295.25 | -303.93 |
| Bootstrapped confidence intervals | | | | |
| min λ | 0.15 | 0.15 | 0.15 | > 0 |
| max λ | 0.25 | 0.25 | 0.20 | |
| min δ | 0.00 | | | |
| max δ | 0.50 | | | |
| Average Max L | -425.96 | -426.01 | -426.30 | -438.90 |

Table 2: Maximum likelihood estimations of the imagination and responsiveness parameters of the EWA learning model.

This table shows the results of the Maximum Likelihood Estimation of the imagination parameter δ , and responsiveness parameter, λ , of the EWA learning model. Initial attractions are set up to fit the behavior of subjects as displayed in the first period and in Moinas and Pouget (2013). Confidence intervals are obtained bootstrapping. 35 estimations are done taking into account the time dependence: 8 estimations after dropping one of the 8 sessions, 1 estimation after dropping the last period; 8 estimations after dropping one of the 8 sessions and the last period; 1 estimation after dropping the last two periods; 8 estimations after dropping one of the 8 sessions and the last two periods; 1 estimation after dropping the last three periods; 8 estimations after dropping one of the 8 sessions and the last three periods. 89% confidence intervals are obtained by dropping the 4 most extreme estimates of each parameter out of the 35 estimations.

6 Conclusion

In this paper, we study whether traders’ experience reduces their propensity to speculate. We theoretically study a financial market populated by adaptive traders. Following Camerer and Ho (1999)’s Experience-Weighted Attraction learning model, these traders are assumed to adjust their behavior according to actions’ past performance. The EWA model nests reinforcement learning, on the one hand, and belief-based learning, according to which agents use their imagination to do counter-factual reasoning, on the other hand.

We focus on the Bubble Game designed by Moinas and Pouget (2013) in which agents sequentially trade a worthless asset. Speculation may arise because agents do not always know where they stand in the market sequence. In the version of the Bubble Game we consider, because there is a cap on the maximum price that can be achieved, no rational bubbles can form.

The learning model shows that, in the long-run, the market converges to the unique no bubble equilibrium. However, we also find that learning may initially increase traders’ propensity to speculate: this is because as long as not all traders have learned not to speculate, some traders can make substantial profit by speculating. In the short run, more experienced traders thus create more bubbles. Moreover, we show that this effect is stronger when traders are more sophisticated and when the price cap is higher.

We provide experimental evidence that is consistent with these theoretical results. First, adaptive learning is at work in speculative markets: their past payoffs have a statistically significant impact on the propensity of subjects to speculate. Secondly, the propensity to speculate and the occurrence of bubbles are reduced with experience when the price cap is low but not when it is high. Overall, our findings reconcile the experimental results of King et al. (1993) and Dufwenberg et al. (2005) who show that experience attenuates the formation of speculative bubbles, and those of Hussam et al. (2008) and Deck et al. (2014) who show that bubbles may occur with experienced traders when market parameters change

and when new traders enter the market.

In future work, it would be interesting to study how traders learn when they can observe other traders' behavior. This would allow one to investigate how observing a bubble inflating affects the propensity to speculate of a trader who has not yet decided to ride the bubble. In the context of the bubble game, this could be captured by enabling agents to observe what the next trader in the market sequence has done. In this context, imagination could play a more important role than in our setting in which it appears not to affect learning.

Appendix A Instructions

INSTRUCTIONS

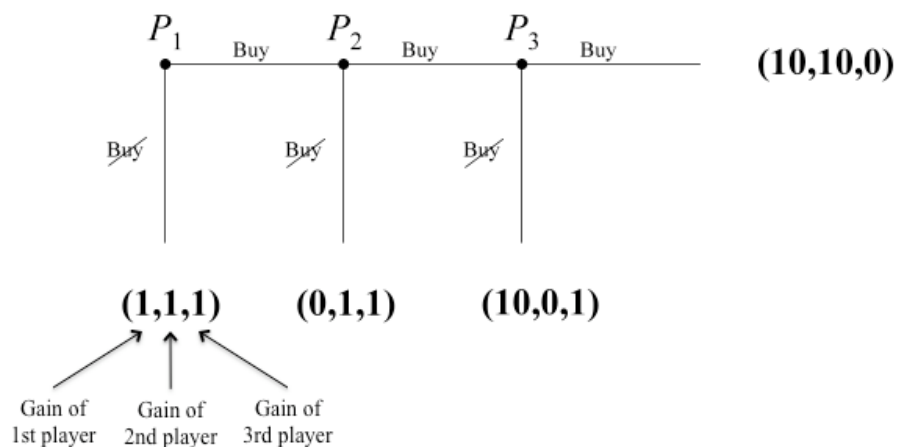
Welcome to this market game. Please read carefully the following instructions. They are identical for all participants. Please do not communicate with the other participants. The game will last approximately one hour, including the reading of the instructions.

You will be able to earn some money during the game. Your total gain will be paid to you on March 27th, 2017.

Trading process

To play this game, we form groups of three players. Each player is endowed with one experimental currency unit (ECU) that can be used to buy an asset. Your task during the game is to decide whether you want to buy or not the asset. This asset does not generate any dividend. If the asset price exceeds 1 ECU, you can still buy the asset. We indeed consider that a financial partner (who is not part of the game) provides you with the additional capital and shares profits with you according to the respective capital invested. The market proceeds sequentially. A proposal to buy at a price P_1 is made to the first player. If he buys, he proposes to sell the asset to the second player at a price which is ten times higher, $P_2 = 10 * P_1$. If the second player accepts to buy, the first player ends up the game with 10 ECU. Otherwise, the first player ends up with nothing. If he or she accepted to buy, the second player then proposes to sell the asset to the third trader at a price $P_3 = 10 * P_2 = 100 * P_1$. If the third player buys the asset, the second player ends up the game with 10 ECU. Otherwise, the second player ends up with nothing. The third player does not find anybody to whom he or she can sell the asset. Since this asset does not generate any dividend, he or she ends up the game with nothing. If a player refuses to buy, he or she ends up the game with 1 ECU (so do all the player after him or her in the market sequence).

At the beginning of the game, players do not know their position in the market sequence. Positions are randomly determined with one chance out of three for each player to be first, second or third. The game is summarized in the following figure.



Proposed prices

The price P_1 that is proposed to the first player is random. This price is a power of 10 and is determined as follows:

| Price P_1 | Probability that this price is realized |
|-------------|---|
| 1 | 1/2 (50%) |
| 10 | 1/4 (25%) |
| 100 | 1/8 (12.5%) |
| 1,000 | 1/16 (6.25%) |
| 10,000 | 1/16 (6.25%) |

Players' decisions are made simultaneously and privately. For example, if the first price $P_1=1$ is drawn, the prices that are simultaneously proposed to the three players are: $P_1=1$ for the first player, $P_2=10$ for the second player, and $P_3=100$ for the third player. Identically, if the first price $P_1=10,000$ has been drawn, the prices that are simultaneously proposed to the three players are: $P_1=10,000$ for the first player, $P_2 = 100,000$ for the second player, and $P_3=1,000,000$ for the third player.

The price that you are been proposed can give you the following information regarding your position in the market sequence:

- if you are proposed to buy at a price of 1, you are sure to be first in the market sequence;
- if you are proposed to buy at a price of 10, you have one chance out of three to be first and two chances out of three to be second in the market sequence. You are sure not to be last;
- if you are proposed to buy at a price of 100 or 1,000, you have one chance out of seven to be first, two chances out of seven to be second, and four chances out seven to be last in the market sequence;
- if you are proposed to buy at a price of 10,000, you have one chance out of four to be first, one chance out of four to be second, and two chances out four to be last in the market sequence;
- if you are proposed to buy at a price of 100,000, you have one chance out of two to be second, and one chance out of two to be third; in this case, you are sure not to be first in the market sequence;
- if you are proposed to buy at a price of 1,000,000, you are sure to be last in the market sequence.

Outcomes and final gain

You are going to participate in 10 independent periods of the game. The composition of each group is anonymous, randomly determined and different at each period of the game. After each period, once decisions are made, we will indicate (anonymously) if you were proposed to buy the asset and at what price. We will also indicate your gain for this period and your overall gain.

At the end of the ten periods of the game, we will compute your total gain as the sum of the gains in each period. Your total gain will be paid to you in euros. The rate of exchange is: 1 € for 1 ECU.

Do you have any question?

QUESTIONNAIRE

- Is it possible to be first and be proposed to buy at a price of 100.000?
 - Yes
 - No
- If you are proposed to buy at a price of 1, are you sure to be first?
 - Yes
 - No
- If you are proposed to buy at a price of 100.000, are you sure to be second?
 - Yes
 - No
- If you are proposed to buy at a price of 100, are you sure to be third?
 - Yes
 - No
- If you are proposed to buy at a price of 1.000.000, are you sure to be third ?
 - Yes
 - No
- If you accept to buy at 100, you will propose to resell at
 - 1.000
 - 1.100
- If you are first, buy at 100, and resell at 1,000, your payoff in the game is
 - 10 euros
 - 900 euros
- If you are first, buy at 100, and find nobody to resell to at 1,000, your payoff in the game is
 - 0 euro
 - 10 euros
- If you refuse to buy, your payoff is
 - 1 euro
 - 0 euro

Appendix B Simulations when a trader observes subsequent trader's behavior

This appendix displays the results of simulations for the same game as in the experiment except that it is assumed that traders can observe the decision of the next trader in the market sequence. Figure 6 refers to the case in which the price cap is 1. Figure 7 refers to the case in which the price cap is 10,000.

When traders observe the actions of the subsequent trader, the propensity to speculate is initially stronger than with no imagination because imaginative traders who are proposed low prices realize that they could have obtained a higher payoff by buying the asset if subsequent traders decided to buy. The speed at which traders learn is then faster.

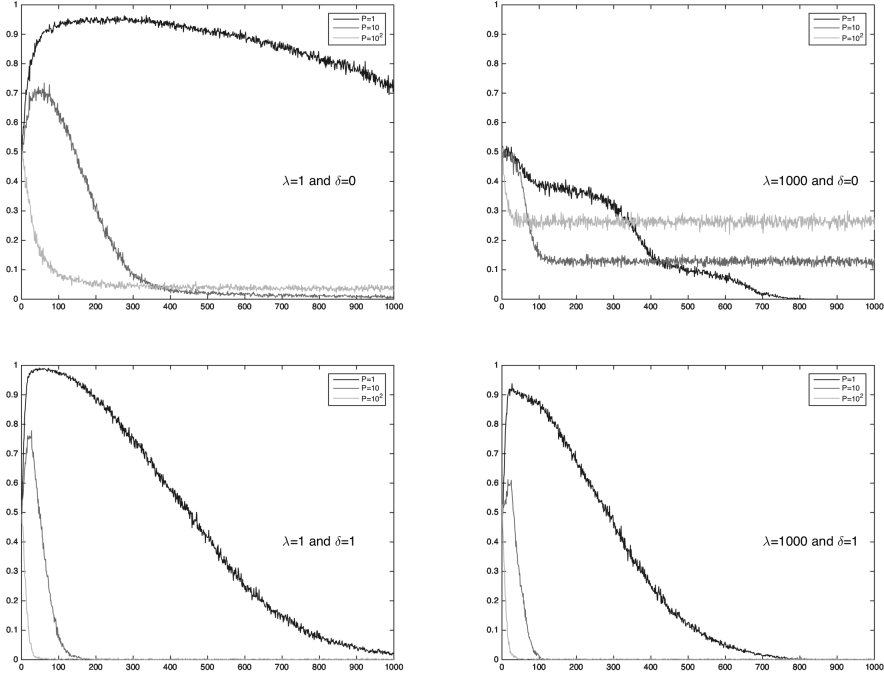


Figure 6: Probability to speculate per price in the EWA learning model when the cap on the first price is 1. Setting in which a trader observes subsequent trader's behavior.

This figure displays the average probability to buy in the Bubble game assuming that traders can observe the choice of the next trader in the market sequence. Behavior is modeled as in the EWA learning model of Camerer and Ho (1999). We set $A_i^j(0|P) = 0$.

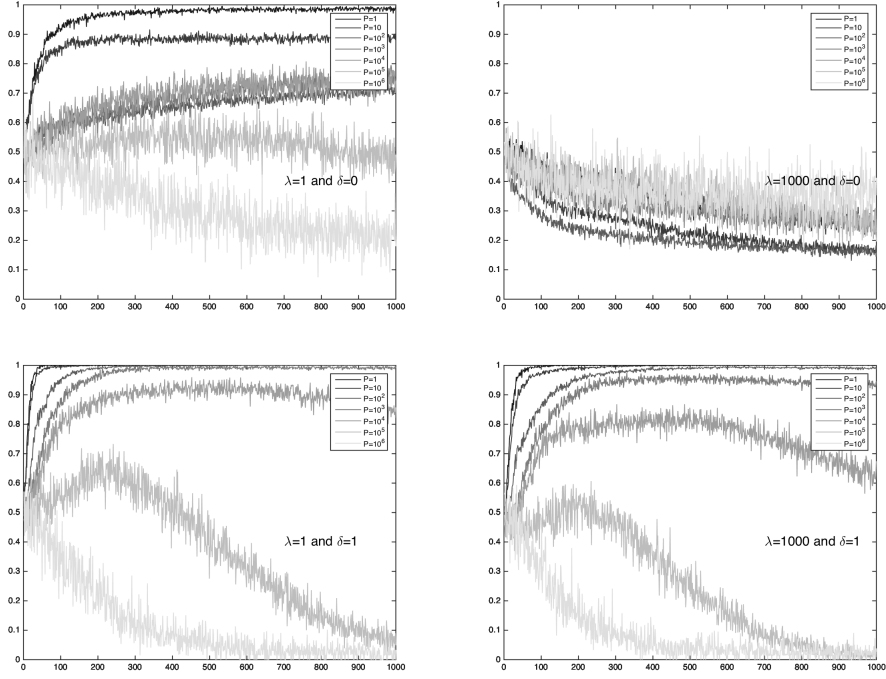


Figure 7: Probability to speculate per price in the EWA learning model when the cap on the first price is 10,000. Setting in which a trader observes subsequent trader's behavior.

This figure displays the average probability to buy in the Bubble game assuming that traders can observe the choice of the next trader in the market sequence. Behavior is modeled as in the EWA learning model of Camerer and Ho (1999). We set $A_i^j(0|P) = 0$.

Appendix C Proofs of the analytical results

Notations – In all the proofs below, $K \in \{0, 4\}$ is the cap on the initial price (i.e., the highest price observed is $P = 10^{K+2}$), $a_i(t)$ is the action chosen by agent i at date t , the set of potential actions j is $\{B, \emptyset\}$ where B stands for a decision to buy, \emptyset for a decision not to buy, $A_i^j(t+1|P)$ is the attraction of action j at period $t+1$ for agent i updated as shown in equation (1), and $Pr_i^j(t+1|P)$ is the probability for agent i to choose action j at period $t+1$ defined in equation (2). Denote $i-$ and $i+$ as the previous and next trader for trader i .

In all the proofs below, we exclude the case in which $\lambda = 0$. In this case indeed, the expected probability to buy is constant and equal to $\frac{1}{2}$ regardless of the price, the period and the past experience.

Proposition 1 – Proof.

We want to show that for any trader i , his probability to buy given any price P will converge to 0 in the long run.

From equation (2), the probability to buy of trader i given price P at period $t+1$ is determined by the difference between his attraction for the action not to buy and that of his action to buy conditional on price P at period t , i.e., $A_i^\emptyset(t|P) - A_i^B(t|P)$.

Applying equation (1) to both actions for trader i , we have:

$$A_i^\emptyset(t|P) - A_i^B(t|P) = A_i^\emptyset(t-1|P) - A_i^B(t-1|P) + \Delta\pi_i(t|P), \quad (3)$$

where we define $\Delta\pi_i(t|P)$ as the incremental payoff between the two actions. Computation yields:

$$\Delta\pi_i(t|P) = \begin{cases} [\delta + (1-\delta)\mathbb{1}_{\emptyset=a_i(t)}] \pi(\emptyset, a_{-i}(t)) - [\delta + (1-\delta)\mathbb{1}_{B=a_i(t)}] \pi(B, a_{-i}(t)) & \text{if } i \text{ observes } P \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Whether the probability to buy of trader i would increase or decrease between t and $t+1$

thus depends on $\Delta\pi_i(t|P)$.

- First, we show that no bubble arises on the highest price, i.e., $P = 10^{K+2}$, in the long run. In the case where $P = 10^{K+2}$, if trader i does not buy, his payoff does not depend on the actions of the others: $\pi(\emptyset, a_{-i}(t)) = 0$. If the trader buys, his payoff depends on the actions of the previous traders; however, given that he observes the highest price, he will never be able to resell the asset, thus $\pi(B, a_{-i}(t)) \in \{0, -1\}$. It follows that $\Delta\pi_i(t|10^{K+2})$ can only take three values, 0, δ or 1:

$$\Delta\pi_i(t|10^{K+2}) = \begin{cases} 1 & \text{trader } i \text{ observes price } 10^{K+2} \text{ and chooses to buy} \\ \delta & \text{trader } i \text{ observes price } 10^{K+2} \text{ and chooses not to buy} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Iterating equation (3) from period 1 to t , and using our assumption that $A_i^\emptyset(0|P) = A_i^B(0|P) = 0$ to initialize the series, the probability to buy at period $t + 1$ conditional on the highest price for trader i can be written as:

$$Pr_i^B(t + 1|10^{K+2}) = \frac{1}{1 + e^{\lambda \sum_{\tau=1}^t \Delta\pi_i(\tau|10^{K+2})}}. \quad (6)$$

From the discussion above, we have $\Delta\pi_i(t|10^{K+2}) \geq 0$.

- Case I: the probability for trader i observing price 10^{K+2} does not converge to 0 as $t \rightarrow +\infty$, i.e., the probability to buy with second and third largest price does not converge to 0. In this case, we suppose that the probability to buy conditional on the highest price does not converge to 0 either. It implies that $\Delta\pi_i(t|10^{K+2}) > 0$ always holds at each period with a non-negligible probability. Thus, $\text{plim}_{t \rightarrow \infty} \sum_{\tau=1}^t \Delta\pi_i(\tau|10^{K+2}) = +\infty$ and $\text{plim}_{t \rightarrow \infty} Pr_i^B(t + 1|10^{K+2}) = 0$. It

contradicts the initial assumption! Thus, the probability to buy given the highest price will converge to 0 as $t \rightarrow +\infty$.

- Case II: the probability for trader i observing price 10^{K+2} converges to 0 when $t \rightarrow +\infty$, i.e., the probability to buy with second or third largest price converges to 0. In this case, trading already stops at the second or third largest price. Thus, no bubble arises with the highest price either.

- Second, we show that no bubble arises on the second highest price, i.e., $P = 10^{K+1}$, in the long run.

- First, we consider the case where the probability to buy given the highest prices converge to 0 in the long run. That is, $\forall \epsilon > 0, \delta > 0$, there exists a T , such that $t > T$, $Pr(|Pr_i^B(t|10^{K+2}) - 0| > \epsilon) < \delta$. We consider the case where $t \geq T$, the probability to buy conditional on the second highest price for any trader j is

$$Pr_j^B(t+1|10^{K+1}) = \frac{1}{1 + e^{-\lambda A_j^B(T-1|10^{K+1})} e^{\lambda \sum_{\tau=T}^t \Delta\pi_i(\tau|10^{K+2})}}, \quad (7)$$

where $e^{-\lambda A_j^B(T-1|10^{K+1})}$ is bounded and $\Delta\pi_i(t|10^{K+1})$ are as follows:

$$\Delta\pi_i(t|10^{K+1}) = \begin{cases} 1 & \text{trader i observes price } 10^{K+1} \text{ and buys but cannot sell} \\ -9 & \text{trader i observes price } 10^{K+1} \text{ and buys and is able to sell} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

- * Case I: the probability for trader j observing price 10^{K+1} does not converge to 0 as $t \rightarrow +\infty$, i.e., the probability to buy with third or fourth largest price does not converge to 0. In this case, we suppose that the probability to buy conditional on the second highest price does not converge to 0 either. It implies that $\Delta\pi_i(t|10^{K+1}) > 0$ happens at each period with a non-negligible probability.

Since $\forall \epsilon > 0, \delta > 0$, there exists a T , such that $t > T$, $Pr(|Pr_i^B(t|10^{K+2}) - 0| > \epsilon) < \delta$. Intuitively, if trader j observes price 10^{K+1} , his probability to sell, i.e., the probability that $\Delta\pi_i(t|10^{K+1}) = -9$, converges to 0 as t is sufficiently large. In summary, the probability that $\Delta\pi_i(t|10^{K+1}) \geq 0$ approaches to 1 and $\Delta\pi_i(t|10^{K+1}) > 0$ happens with a non-negligible probability at each period when t is sufficiently large. Thus, $\text{plim}_{t \rightarrow \infty} \sum_{\tau=T}^t \Delta\pi_i(\tau|10^{K+1}) = +\infty$. Since $e^{-\lambda A_j^B(T-1|10^{K+1})}$ is bounded, we obtain that $\text{plim}_{t \rightarrow \infty} Pr_i^B(t+1|10^{K+1}) = 0$. It contradicts the initial assumption! Thus, the probability to buy given the second highest price converges to 0 in the long run.

* Case II: the probability for trader j observing price 10^{K+1} converges to 0 when $t \rightarrow +\infty$, i.e., the probability to buy with third or fourth largest price converges to 0. In this case, trading already stops at the third or second largest price. Thus, no bubble arises with the second highest price either.

– Second, we consider the case where no bubble arises for the highest price because that trading already stops at the second or third largest price. It is obvious that, in this case, no bubble arises with the second highest price either.

- Last, according to the same reasoning, the same analysis can be applied to the traders with other lower prices and we can show that no bubble arises conditional on those prices. Therefore, in the long run, the trading should converge to no bubble equilibrium.

■

Proposition 2 – Proof.

At period 1, the probability to buy for any trader at any price is $\frac{1}{2}$ since the initial attractions are assumed to be 0. Let us analyze whether the probability to buy of a trader i at period 2 for any price $P \in \{1, \dots, 10^k, \dots, 10^{K+2}\}$ is higher or smaller than $\frac{1}{2}$.

Let us denote by q_r^P the probability for a trader who is in position $r \in \{1, 2, 3\}$ in the market sequence to observe price P . More precisely, $q_1^P = Pr(\text{First observes } P)$, $q_2^P = Pr(\text{Second observes } P)$, and $q_3^P = Pr(\text{Third observes } P)$. We have:

$$q_1^{10^k} = q_2^{10^{k+1}} = q_3^{10^{k+2}} = \begin{cases} \frac{1}{2}^{k+1} & \text{if } k < K \\ \frac{1}{2}^K & \text{if } k = K \\ 0 & \text{if } k > K. \end{cases} \quad (9)$$

At period 1, there are four different potential cases. Trader i will observe price P with probability $\frac{1}{3}(q_1^P + q_2^P + q_3^P)$. His incremental payoff between not buy and buy thus writes as follows:

- With probability $Pr(\text{First observes } P \cap i \text{ is First}) = \frac{1}{3}q_1^P$, trader i observes price P and is in first position. In this case,

$$\Delta\pi_i(1|P) = \begin{cases} -9 & \text{if } a_i(1) = B \text{ and } a_{i+}(1) = B & (\frac{1}{4}) \\ 1 & \text{if } a_i(1) = B \text{ and } a_{i+}(1) = \emptyset & (\frac{1}{4}) \\ 0 & \text{if } a_i(1) = \emptyset & (\frac{1}{2}) \end{cases} \quad (10)$$

If trader i buys the asset and the next trader also buys the asset, which happens with probability $\frac{1}{4}$, the incremental payoff the trader i can obtain between not buy and buy is -9 . If trader i buys the asset but the next trader refuses to buy, which happens with probability $\frac{1}{4}$, the incremental payoff is 1. Finally, if trader i refuses to buy, which happens with probability $\frac{1}{2}$, the incremental payoff is 0. In the latter case, imagination for what would happen if he had bought would not kick in since trader i is not provided the information on the next trader.

- With probability $Pr(\text{Second observes } P \cap i \text{ is Second}) = \frac{1}{3}q_2^P$, trader i observes price P and is in second position. This yields:

$$\Delta\pi_i(1|P) = \begin{cases} 0 & \text{if } a_{i-}(1) = \emptyset & \left(\frac{1}{2}\right) \\ -9 & \text{if } a_{i-}(1) = B \text{ and } a_i(1) = B \text{ and } a_{i+}(1) = B & \left(\frac{1}{8}\right) \\ 1 & \text{if } a_{i-}(1) = B \text{ and } a_i(1) = B \text{ and } a_{i+}(1) = \emptyset & \left(\frac{1}{8}\right) \\ 0 & \text{if } a_{i-}(1) = B \text{ and } a_i(1) = \emptyset & \left(\frac{1}{4}\right) \end{cases} \quad (11)$$

In this case, the incremental payoff not only depends on the action of the next trader, but also on that of the previous one. If the previous trader chooses not to buy, which happens with probability $\frac{1}{2}$, the incremental payoff is 0; otherwise, the three cases are similar to those described below equation (10).

- With probability $Pr(\text{Third observes } P \cap i \text{ is Third}) = \frac{1}{3}q_3^P$, trader i observes price P and is in third position. Following the same reasoning as above, if $P < 10^{K+2}$, we have:

$$\Delta\pi_i(1|P) = \begin{cases} 1 & \text{if } a_i(1) = B \text{ and } a_{i-}(1) = B \text{ and } a_{i--}(1) = B & \left(\frac{1}{8}\right) \\ 0 & \text{otherwise} & \left(\frac{7}{8}\right) \end{cases} \quad (12)$$

In this case, the incremental payoff depends on not only the action of trader i but also the actions of the previous two traders. If the previous two traders and trader i choose to buy, which happens with probability $\frac{1}{8}$, the incremental payoff is 1; otherwise, the incremental payoff is 0.

The case in which trader i observes the highest possible price, i.e., $P = 10^{K+2}$ is slightly different: in that case indeed, a trader who would not buy could still imagine the payoff he would have received if he had bought given that he infers from P his position in the market sequence. This yields:

$$\Delta\pi_i(1|10^{K+2}) = \begin{cases} 1 & \text{if } a_i(1) = B \text{ and } a_{i-}(1) = B \text{ and } a_{i--}(1) = B & (\frac{1}{8}) \\ \delta & \text{if } a_i(1) = \emptyset \text{ and } a_{i-}(1) = B \text{ and } a_{i--}(1) = B & (\frac{1}{8}) \\ 0 & \text{otherwise} & (\frac{3}{4}) \end{cases} \quad (13)$$

In this case, the incremental payoff is 1 if all the three traders choose to buy, and δ if the first two traders choose to buy and trader i chooses not to buy since he can imagine that he would have received -1 if he had bought. For other cases, the incremental payoff is 0.

- With probability $1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P)$, trader i does not observe price P and $\Delta\pi_i(1|P) = 0$.

To sum up, the incremental payoff trader i can obtain between not buy and buy is as follows: for price $P < 10^{K+2}$

$$\Delta\pi_i(1|P) = \begin{cases} 0 & \text{with prob. } 1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P) + \frac{1}{6}(q_1^P + \frac{3}{2}q_2^P + \frac{7}{4}q_3^P) \\ 1 & \text{with prob. } \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P + \frac{1}{2}q_3^P) \\ -9 & \text{with prob. } \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P), \end{cases} \quad (14)$$

and for the highest possible price $P = 10^{K+2}$:

$$\Delta\pi_i(1|10^{K+2}) = \begin{cases} 0 & \text{with prob. } 1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P) + \frac{1}{6}(q_1^P + \frac{3}{2}q_2^P + \frac{3}{2}q_3^P) \\ 1 & \text{with prob. } \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P + \frac{1}{2}q_3^P) \\ -9 & \text{with prob. } \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P) \\ \delta & \text{with prob. } \frac{1}{24}q_3^P \end{cases} \quad (15)$$

We now compared the probability to buy at period 2 with the probability to buy at period 1 which is $\frac{1}{2}$. From equation (2), the probability to buy for trader i conditional on price P at period 2 can be written as:

$$Pr_i^B(2|P) = \frac{1}{1 + e^{\lambda \Delta \pi_i(1|P)}}, \quad (16)$$

where $\Delta \pi_i(1|P)$ is distributed according to equation (14) if $P < 10^{K+2}$ and equation (15) if $P = 10^{K+2}$.

We can see that the expected probability to buy at period 2 for trader i if $P < 10^{K+2}$ is

$$\begin{aligned} E_1(Pr_i^B(2|P)) &= \left[1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P) + \frac{1}{6}(q_1^P + \frac{3}{2}q_2^P + \frac{7}{4}q_3^P) \right] \times \frac{1}{2} \\ &\quad + \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P + \frac{1}{2}q_3^P) \times \frac{1}{1 + e^\lambda} \\ &\quad + \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P) \times \frac{1}{1 + e^{-9\lambda}}, \end{aligned} \quad (17)$$

and the expected probability to buy at period 2 for trader i if $P = 10^{K+2}$ is

$$\begin{aligned} E_1(Pr_i^B(2|10^{K+2})) &= \left[1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P) + \frac{1}{6}(q_1^P + \frac{3}{2}q_2^P + \frac{3}{2}q_3^P) \right] \times \frac{1}{2} \\ &\quad + \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P + \frac{1}{2}q_3^P) \times \frac{1}{1 + e^\lambda} \\ &\quad + \frac{1}{12}(q_1^P + \frac{1}{2}q_2^P) \times \frac{1}{1 + e^{-9\lambda}} \\ &\quad + \frac{1}{24}q_3^P \frac{1}{1 + e^{\lambda\delta}}. \end{aligned} \quad (18)$$

Finally, the expected probability can be simplified as follows:

$$E_1(Pr_i^B(2|P)) = \begin{cases} \frac{1}{12} \left[6 - q_1^P - \frac{q_2^P}{2} - \frac{q_3^P}{4} + \frac{q_1^P + q_2^P}{1 + e^{-9\lambda}} + \frac{2q_1^P + q_2^P + q_3^P}{2(1 + e^\lambda)} \right] & \text{if } P < 10^{K+2} \\ \frac{1}{24} \left[12 + (2q_1^P + q_2^P) \left(\frac{1}{1 + e^\lambda} + \frac{1}{1 + e^{-9\lambda}} - 1 \right) + q_3^P \left(\frac{1}{1 + e^\lambda} + \frac{1}{1 + e^{\lambda\delta}} - 1 \right) \right] & \text{if } P = 10^{K+2}. \end{cases} \quad (19)$$

It is useful to compute the probabilities q_r^P for $r \in \{1, 2, 3\}$ in equation (9) explicitly. Tables 3 and 4 show the values of these probabilities for any price P when the cap on the

Table 3: Price Cap on the first price is 1

| Price P | q_1^P | q_2^P | q_3^P | $E_1(Pr_i^B(2 P))$ |
|---------|---------|---------|---------|--|
| 1 | 1 | 0 | 0 | $\frac{1}{12} \left(5 + \frac{1}{1+e^{-9\lambda}} + \frac{1}{1+e^\lambda} \right)$ |
| 10 | 0 | 1 | 0 | $\frac{1}{24} \left(11 + \frac{1}{1+e^{-9\lambda}} + \frac{1}{1+e^\lambda} \right)$ |
| 100 | 0 | 0 | 1 | $\frac{1}{24} \left(11 + \frac{1}{1+e^\lambda} + \frac{1}{1+e^{\delta\lambda}} \right)$ |

Table 4: Price Cap on the first price is 10^4

| Price P | q_1^P | q_2^P | q_3^P | $E_1(Pr_i^B(2 P))$ |
|---------|----------------|----------------|----------------|--|
| 1 | $\frac{1}{2}$ | 0 | 0 | $\frac{1}{24} \left(11 + \frac{1}{1+e^{-9\lambda}} + \frac{1}{1+e^\lambda} \right)$ |
| 10 | $\frac{1}{4}$ | $\frac{1}{2}$ | 0 | $\frac{1}{24} \left(11 + \frac{1}{1+e^{-9\lambda}} + \frac{1}{1+e^\lambda} \right)$ |
| 10^2 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{24} \left(\frac{45}{4} + \frac{1}{2} \frac{1}{1+e^{-9\lambda}} + \frac{1}{1+e^\lambda} \right)$ |
| 10^3 | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{192} \left(93 + \frac{2}{1+e^{-9\lambda}} + \frac{4}{1+e^\lambda} \right)$ |
| 10^4 | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{384} \left(188 + \frac{3}{1+e^{-9\lambda}} + \frac{5}{1+e^\lambda} \right)$ |
| 10^5 | 0 | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{768} \left(381 + \frac{2}{1+e^{-9\lambda}} + \frac{4}{1+e^\lambda} \right)$ |
| 10^6 | 0 | 0 | $\frac{1}{16}$ | $\frac{1}{384} \left(191 + \frac{1}{1+e^\lambda} + \frac{1}{1+e^{\delta\lambda}} \right)$ |

first price are 1 and 10^4 respectively. Notice that $0 \leq q_1^P + q_2^P + q_3^P \leq 1$.

- First, we analyze the case where trader i observes the highest price. Notice that $q_1^{10^{K+2}} = q_2^{10^{K+2}} = 0$ and $q_3^{10^{K+2}} > 0$. If trader i observes the highest price, we obtain from equation (19) that

$$E_1(Pr_i^B(2|10^{K+2})) = \frac{1}{24} \left[12 + q_3^{10^{K+2}} \left(\frac{1}{1+e^\lambda} + \frac{1}{1+e^{\delta\lambda}} - 1 \right) \right],$$

which is strictly lower than $\frac{1}{24} \left[12 + q_3^{10^{K+2}} \left(\frac{1}{2} + \frac{1}{2} - 1 \right) \right] = \frac{1}{2}$ for $\lambda > 0$ and $\delta \geq 0$. The probability to buy of a trader who observes the highest price is expected to decrease immediately between period 1 and period 2.

- Second, let us analyze the cases where trader i observes the two lowest prices, $P = 1$ or $P = 10$. Notice that trader i can never be last, i.e., $q_3^1 = q_3^{10} = 0$. It follows from equation (19) that:

$$E_1(Pr_i^B(2|P)) = \frac{1}{12} \left[6 - q_1^P - \frac{1}{2}q_2^P + (q_1^P + \frac{q_2^P}{2}) \left(\frac{1}{1+e^{-9\lambda}} + \frac{1}{1+e^\lambda} \right) \right].$$

Now, we have if $\lambda > 0$:

$$\frac{1}{1+e^{-9\lambda}} + \frac{1}{1+e^\lambda} = 1 - \frac{1}{1+e^{9\lambda}} + \frac{1}{1+e^\lambda} > 1 - \frac{1}{1+e^\lambda} + \frac{1}{1+e^\lambda} = 1$$

Based on the above two results, we can see that the expected probability to buy at period 2 when $P \in \{1, 10\}$ satisfies

$$E_1(Pr_i^B(2|P)) > \frac{1}{12} \left[6 - q_1^P - \frac{1}{2}q_2^P + (q_1^P + \frac{q_2^P}{2}) \right] = \frac{1}{2}$$

Therefore, the expected probability to buy increases for prices 1 and 10 at period 2 whatever the cap on the initial price.

- Third, when the cap is $K = 4$, let us analyze the cases where trader i observes the price is 10^k , where $k = 2, 3, 4, 5$. To this end, we differentiate the expected probability to buy defined in equation (19) with respect to λ , using the exact values of q_r^P defined

in Table 4. Computation yields:

$$\begin{aligned}
\frac{\partial E_1(Pr_i^B(2|10^2))}{\partial \lambda} &= \frac{1}{48} e^\lambda \left(\frac{9e^{8\lambda}}{(1+e^{9\lambda})^2} - \frac{2}{(1+e^\lambda)^2} \right) \\
\frac{\partial E_1(Pr_i^B(2|10^3))}{\partial \lambda} &= \frac{1}{96} e^\lambda \left(\frac{9e^{8\lambda}}{(1+e^{9\lambda})^2} - \frac{2}{(1+e^\lambda)^2} \right) \\
\frac{\partial E_1(Pr_i^B(2|10^4))}{\partial \lambda} &= \frac{1}{384} e^\lambda \left(\frac{27e^{8\lambda}}{(1+e^{9\lambda})^2} - \frac{5}{(1+e^\lambda)^2} \right) \\
\frac{\partial E_1(Pr_i^B(2|10^5))}{\partial \lambda} &= \frac{2}{768} e^\lambda \left(\frac{9e^{8\lambda}}{(1+e^{9\lambda})^2} - \frac{2}{(1+e^\lambda)^2} \right)
\end{aligned}$$

It is easy to show that the first order differentiation is strictly positive if and only if $0 \leq \lambda < \lambda^*(P)$, where $\lambda^*(P) \in (0, +\infty)$. That is, for all $P \in \{10^2, 10^3, 10^4, 10^5\}$, there exists a $\lambda^*(P) \in (0, +\infty)$ such that the expected probability to buy at price P increases with λ when $\lambda \leq \lambda^*(P)$ and decreases with λ when $\lambda > \lambda^*(P)$.

When $\lambda = 0$, the expected probability to buy at period 2 is constant and equal to $\frac{1}{2}$ regardless of the price. When $\lambda \rightarrow +\infty$, the expected probability to buy at period 2 are $\frac{47}{96}$, $\frac{95}{192}$, $\frac{191}{384}$ and $\frac{383}{768}$ at prices 10^2 , 10^3 , 10^4 and 10^5 respectively. These probabilities are all lower than $\frac{1}{2}$. Therefore, there exists a $\lambda'(P) \in (0, +\infty)$ such that the expected probability to buy at period 2 is greater than $\frac{1}{2}$ if $\lambda \in (0, \lambda'(P))$ and lower than $\frac{1}{2}$ if $\lambda \in (\lambda'(P), +\infty)$.

■

Appendix D Traders' speculation in the short term given the initial probabilities from the ex- perimental data

Notations – In all the proofs below, $Pr_i^B(t|P, K)$ represents the probability to buy for trader i at period t conditional on price P in the case where the price cap on the first price is 10^K . $A_i^j(t|P, K)$ represents the attraction of trader i at period t conditional on price P in the case where the price cap on the first price is 10^K , where $j \in \{B, \emptyset\}$ and $K \in \{0, 4\}$.

Proof.

From the first replications of the experiment, we know that: in the case where $K = 0$, the probabilities to buy at first period $Pr_i^B(1|P, 0)$ are 80.00%, 55.00% and 10.00% when the price P is 1, 10, and 100 respectively. In the case where $K = 4$, the probabilities to buy at first period $Pr_i^B(1|P, 4)$ are 95.24%, 94.29%, 62.50%, 50.00%, 60.00%, 20.00% and 10.00% when price P is 1, 10, ..., 10^5 and 10^6 respectively. We analyze whether the probability to buy of a trader i at period 2 will increase or decrease relative to that at period 1.

At period 1, there are four different potential cases. Trader i observes price P with probability $\frac{1}{3}(q_1^P + q_2^P + q_3^P)$, where q_1^P , q_2^P and q_3^P satisfy equation (9). The attraction of trader i at period 2 is updated as follows:

- With probability $Pr(\text{First observes } P \cap i \text{ is First}) = \frac{1}{3}q_1^P$, trader i observes price P and is in first position. In this case,

$$A_i^B(1|P, K) = A_i^B(0|P, K) + \begin{cases} 9 & \text{if } a_i(1) = a_{i+}(1) = B & (b_1) \\ -1 & \text{if } a_i(1) = B \text{ and } a_{i+}(1) = \emptyset & (b_2) \\ 0 & \text{if } a_i(1) = \emptyset & (b_3) \end{cases} \quad (20)$$

where

$$b_1 = Pr_i^B(1|P, K)Pr_i^B(1|10P, K),$$

$$b_2 = Pr_i^B(1|P, K) (1 - Pr_i^B(1|10P, K)),$$

$$\text{and } b_3 = 1 - Pr_i^B(1|P, K).$$

If trader i buys the asset and the next trader also buys the asset, which happens with probability b_1 , the attraction of buying for trader i will add profit 9. If trader i buys the asset but the next trader refuses to buy, which happens with probability b_2 , the attraction will add loss -1 . If trader i refuses to buy, which happens with probability b_3 , the attraction will be exactly the same as the initial one. In the latter case, imagination for what would happen if he had bought would not kick in since trader i is not provided the information on the next trader.

- With probability $Pr(\text{Second observes } P \cap i \text{ is Second}) = \frac{1}{3}q_2^P$, trader i observes price P and is in second position. This yields:

$$A_i^B(1|P, K) = A_i^B(0|P, K) + \begin{cases} 0 & \text{if } a_{i-}(1) = \emptyset & (b_4) \\ 9 & \text{if } a_{i-}(1) = a_i(1) = a_{i+}(1) = B & (b_5) \\ -1 & \text{if } a_{i-}(1) = a_i(1) = B \text{ and } a_{i+}(1) = \emptyset & (b_6) \\ 0 & \text{if } a_{i-}(1) = B \text{ and } a_i(1) = \emptyset & (b_7) \end{cases} \quad (21)$$

where

$$b_4 = 1 - Pr_i^B(1|\frac{P}{10}, K),$$

$$b_5 = Pr_i^B(1|\frac{P}{10}, K)Pr_i^B(1|P, K)Pr_i^B(1|10P, K),$$

$$b_6 = Pr_i^B(1|\frac{P}{10}, K)Pr_i^B(1|P, K) (1 - Pr_i^B(1|10P, K))$$

$$\text{and } b_7 = Pr_i^B(1|\frac{P}{10}, K) (1 - Pr_i^B(1|P, K)).$$

In this case, the attraction not only depends on the action of the next trader, but also on that of the previous one. If the previous trader chooses not to buy, which happens with

probability b_4 , the attraction is exactly the same as the initial attractions; otherwise, the three cases are similar to those described below equation (20).

- With probability $Pr(\text{Third observes } P \cap i \text{ is Third}) = \frac{1}{3}q_3^P$, trader i observes price P and is in third position. Following the same reasoning as above, if $P < 10^{K+2}$, we have:

$$A_i^B(1|P, K) = A_i^B(0|P, K) + \begin{cases} -1 & \text{if } a_i(1) = a_{i-}(1) = a_{i--}(1) = B & (b_8) \\ 0 & \text{otherwise} & (b_9) \end{cases} \quad (22)$$

where

$$b_8 = Pr_i^B(1|\frac{P}{100}, K)Pr_i^B(1|\frac{P}{10}, K)Pr_i^B(1|P, K),$$

$$\text{and } b_9 = 1 - Pr_i^B(1|\frac{P}{100}, K)Pr_i^B(1|\frac{P}{10}, K)Pr_i^B(1|P, K).$$

In this case, the attraction depends on not only the action of trader i but also the actions of the previous two traders. If all the three traders choose to buy, which happens with probability b_8 , the attraction will add -1 ; otherwise, the attraction is exactly the same as the initial attraction.

The case in which trader i observes the highest possible price, i.e., $P = 10^{K+2}$ is slightly different: in that case indeed, a trader who would not buy could still imagine the payoff he would have received if he had bought given that he infers from P his position in the market sequence. This yields:

$$A_i^B(1|10^{K+2}, K) = A_i^B(0|10^{K+2}, K) + \begin{cases} -1 & \text{if } a_i(1) = a_{i-}(1) = a_{i--}(1) = B & (b_{10}) \\ -\delta & \text{if } a_i(1) = \emptyset \text{ and } a_{i-}(1) = a_{i--}(1) = B & (b_{11}) \\ 0 & \text{otherwise} & (b_{12}) \end{cases} \quad (23)$$

where

$$b_{10} = Pr_i^B(1|\frac{P}{100}, K)Pr_i^B(1|\frac{P}{10}, K)Pr_i^B(1|P, K),$$

$$b_{11} = Pr_i^B(1|\frac{P}{100}, K)Pr_i^B(1|\frac{P}{10}, K) (1 - Pr_i^B(1|P, K)),$$

and $b_{12} = 1 - Pr_i^B(1|\frac{P}{100}, K)Pr_i^B(1|\frac{P}{10}, K)$.

In this case, the attraction will add -1 if all the three traders choose to buy, and $-\delta$ if the first two traders choose to buy and trader i chooses not to buy since he can imagine that he would have received -1 if he had bought. For other cases, the incremental payoff is 0.

- With probability $1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P)$, trader i does not observe price P and $A_i^B(1|P, K) = A_i^B(0|P, K)$.

To sum up, the attraction of buying for trader i is as follows: if price $P < 10^{K+2}$, we have

$$A_i^B(1|P, K) = A_i^B(0|P, K) + \begin{cases} 0 & \text{with prob. } c_1 \\ -1 & \text{with prob. } c_2 \\ 9 & \text{with prob. } c_3, \end{cases} \quad (24)$$

where

$$\begin{aligned} c_1 &= \frac{1}{3}q_1^P b_3 + \frac{1}{3}q_2^P (b_4 + b_7) + \frac{1}{3}q_3^P b_9 + 1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P), \\ c_2 &= \frac{1}{3}q_1^P b_2 + \frac{1}{3}q_2^P b_6 + \frac{1}{3}q_3^P b_8, \\ c_3 &= \frac{1}{3}q_1^P b_1 + \frac{1}{3}q_2^P b_5. \end{aligned} \quad (25)$$

If $P = 10^{K+2}$, we have

$$A_i^B(1|10^{K+2}, K) = A_i^B(0|10^{K+2}, K) + \begin{cases} 0 & \text{with prob. } c_4 \\ -1 & \text{with prob. } c_5 \\ 9 & \text{with prob. } c_6 \\ -\delta & \text{with prob. } c_7. \end{cases} \quad (26)$$

where

$$\begin{aligned}
c_4 &= \frac{1}{3}q_1^P b_3 + \frac{1}{3}q_2^P (b_4 + b_7) + \frac{1}{3}q_3^P b_{12} + 1 - \frac{1}{3}(q_1^P + q_2^P + q_3^P), \\
c_5 &= \frac{1}{3}q_1^P b_2 + \frac{1}{3}q_2^P b_6 + \frac{1}{3}q_3^P b_{10}, \\
c_6 &= \frac{1}{3}q_1^P b_1 + \frac{1}{3}q_2^P b_5, \\
c_7 &= \frac{1}{3}q_3^P b_{11}.
\end{aligned} \tag{27}$$

From equation (2), the probability to buy for trader i conditional on price P at period 2 is:

$$Pr_i^B(2|P, K) = \frac{1}{1 + e^{-\lambda A_i^B(1|P, K)}}, \tag{28}$$

where $A_i^B(1|P, K)$ is distributed according to equation (24) if $P < 10^{K+2}$ and equation (26) if $P = 10^{K+2}$.

We obtain that $e^{-\lambda A_i^B(0|P, K)} = \frac{1}{Pr_i^B(1|P, K)} - 1$. We can see that the expected probability to buy at period 2 for trader i is computed as follows: if $P < 10^{K+2}$, we have

$$E_1(Pr_i^B(2|P, K)) = c_1 Pr_i^B(1|P, K) + \frac{c_2}{1 + e^{\lambda \left(\frac{1}{Pr_i^B(1|P, K)} - 1 \right)}} + \frac{c_3}{1 + e^{-9\lambda \left(\frac{1}{Pr_i^B(1|P, K)} - 1 \right)}}, \tag{29}$$

where c_1 , c_2 and c_3 are as equation (25).

If $P = 10^{K+2}$, we have

$$\begin{aligned}
E_1(Pr_i^B(2|10^{K+2}, K)) &= c_4 Pr_i^B(1|P, K) + \frac{c_5}{1 + e^{\lambda \left(\frac{1}{Pr_i^B(1|P, K)} - 1 \right)}} \\
&\quad + \frac{c_6}{1 + e^{-9\lambda \left(\frac{1}{Pr_i^B(1|P, K)} - 1 \right)}} + \frac{c_7}{1 + e^{\delta\lambda \left(\frac{1}{Pr_i^B(1|P, K)} - 1 \right)}},
\end{aligned} \tag{30}$$

where c_4 , c_5 , c_6 and c_7 are as equation (27).

Tables 5 and 6 show the values of these probabilities for any price P when the cap on

Table 5: Price Cap on the first price is 1

| Price P | q_1^P | q_2^P | q_3^P | $E_1(Pr_i^B(2 P, K = 1))$ |
|---------|---------|---------|---------|---|
| 1 | 1 | 0 | 0 | $\frac{1}{75} \left[44 + \frac{44}{4+e^{-9\lambda}} + \frac{36}{4+e^\lambda} \right]$ |
| 10 | 0 | 1 | 0 | $\frac{11}{750} \left[32 + \frac{11}{11+9e^{-9\lambda}} + \frac{99}{11+9e^\lambda} \right]$ |
| 100 | 0 | 0 | 1 | $\frac{1}{750} \left[64 + \frac{11}{1+9e^\lambda} + \frac{99}{1+9e^{\delta\lambda}} \right]$ |

Table 6: Price Cap on the first price is 10^4

| Price P | q_1^P | q_2^P | q_3^P | $E_1(Pr_i^B(2 P, K = 4))$ |
|---------|----------------|----------------|----------------|--|
| 1 | $\frac{1}{2}$ | 0 | 0 | $\frac{2381}{1.5 \times 10^8} \left[50476 + \frac{2381 \times 9429}{2381+119e^{-9\lambda}} + \frac{571 \times 2381}{2381+119e^\lambda} \right]$ |
| 10 | $\frac{1}{4}$ | $\frac{1}{2}$ | 0 | $\frac{3143}{5 \times 10^{11}} \left[115763301 + \frac{22693750 \times 9429}{9429+571e^{-9\lambda}} + \frac{13616250 \times 9429}{9429+571e^\lambda} \right]$ |
| 10^2 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{1.92 \times 10^9} \left[997566505 + \frac{360725000}{5+3e^{-9\lambda}} + \frac{1258742960}{5+3e^\lambda} \right]$ |
| 10^3 | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{768000} \left[360571 + \frac{16800}{1+e^{-9\lambda}} + \frac{30058}{1+e^\lambda} \right]$ |
| 10^4 | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{8}$ | $\frac{3}{3200} \left[623 + \frac{12}{3+2e^{-9\lambda}} + \frac{73}{3+2e^\lambda} \right]$ |
| 10^5 | 0 | $\frac{1}{16}$ | $\frac{1}{16}$ | $\frac{1}{4000} \left(797 + \frac{1}{1+4e^{-9\lambda}} + \frac{14}{1+4e^\lambda} \right)$ |
| 10^6 | 0 | 0 | $\frac{1}{16}$ | $\frac{1}{4000} \left[399 + \frac{1}{1+9e^\lambda} + \frac{9}{1+9e^{\delta\lambda}} \right]$ |

the first price are 1 and 10^4 respectively. Notice that $0 \leq q_1^P + q_2^P + q_3^P \leq 1$.

- First, let us analyze the case where trader i observes the highest or the second highest prices.

– Case I: $K = 0$.

In this case, $q_1^{100} = q_2^{100} = 0$ and $q_3^{100} = 1$. Based on the initial probabilities to buy and equation (30), we obtain the expected probability to buy at period 2 if trader i observes price 100 as follows:

$$E_1(Pr_i^B(2|100, 0)) = \frac{1}{750} \left[64 + \frac{11}{1+9e^\lambda} + \frac{99}{1+9e^{\delta\lambda}} \right],$$

which is strictly lower than 0.1 for $\lambda > 0$ and $\delta \geq 0$. The probability to buy of a trader who observes the highest price 100 when the price cap is 1 is expected to decrease immediately between period 1 and period 2.

We look at the second highest price 10. $q_1^{10} = q_3^{10} = 0$ and $q_2^{10} = 1$. Given the initial probabilities and equation (29), similarly we can show that the expected probability to buy at the second highest price 10

$$E_1(Pr_i^B(2|10, 0)) = \frac{11}{750} \left[32 + \frac{11}{11 + 9e^{-9\lambda}} + \frac{99}{11 + 9e^{\lambda}} \right],$$

which is strictly lower than 0.55 (the probability to buy at period 1) when $\lambda > 0$ and $\delta \geq 0$.

– Case II: $K = 4$.

In this case, $q_1^{10^6} = q_2^{10^6} = 0$ and $q_3^{10^6} = \frac{1}{16}$. Given the initial probabilities and equation (30), we also obtain the expected probability to buy at period 2 is

$$E_1(Pr_i^B(2|10^6, K)) = \frac{1}{4000} \left[399 + \frac{1}{1 + 9e^{\lambda}} + \frac{9}{1 + 9e^{\delta\lambda}} \right],$$

which is strictly lower than 0.1 for $\lambda > 0$ and $\delta \geq 0$. The probability to buy of a trader who observes the highest price 10^6 when the price cap is 10^4 is expected to decrease immediately between period 1 and period 2.

We look at the second highest price 10^5 , where $q_1^{10^5} = 0$ and $q_2^{10^5} = q_3^{10^5} = \frac{1}{16}$. Given the initial probabilities and equation (29)), similarly we can show that the probability to buy at the second highest price 10^5

$$E_1(Pr_i^B(2|10^5, K)) = \frac{1}{4000} \left(797 + \frac{1}{1 + 4e^{-9\lambda}} + \frac{14}{1 + 4e^{\lambda}} \right),$$

which is strictly lower than 0.2 (the probability to buy at period 1) when $\lambda > 0$ and $\delta \geq 0$.

– In summary, the probability to buy of a trader who observes the highest or the second highest price is expected to decrease between period 1 and period 2.

- Second, we analyze the cases where trader i observes prices which are not the highest or second highest to check whether they will have incentive to speculate more next period, i.e., period 2. We compute the expected probability to buy at different prices given different caps by using equation (29). Similarly, we obtain the expected probabilities to buy in Tables 5 and 6. To this end, we differentiate the expected probability to buy in these two tables with respect to λ . Computation yields:

$$\begin{aligned}
\frac{\partial E_1(Pr_i^B(2|1,0))}{\partial \lambda} &= \frac{1}{75} \left(\frac{396e^{9\lambda}}{(1+4e^{9\lambda})^2} - \frac{36e^\lambda}{(4+e^\lambda)^2} \right) \\
\frac{\partial E_1(Pr_i^B(2|1,4))}{\partial \lambda} &= \frac{283339e^{-9\lambda}}{1.5 \times 10^8} \left(\frac{84861}{(2381+119e^{-9\lambda})^2} - \frac{571e^{10\lambda}}{(2381+119e^\lambda)^2} \right) \\
\frac{\partial E_1(Pr_i^B(2|10,4))}{\partial \lambda} &= \frac{184328983711341e^{-9\lambda}}{400000000} \left(\frac{15}{(9429+571e^{-9\lambda})^2} - \frac{e^{10\lambda}}{(9429+571e^\lambda)^2} \right) \\
\frac{\partial E_1(Pr_i^B(2|10^2,4))}{\partial \lambda} &= \frac{e^\lambda}{1.6 \times 10^7} \left(\frac{81163125e^{8\lambda}}{(3+e^{9\lambda})^2} - \frac{31468574}{(5+3e^\lambda)^2} \right) \\
\frac{\partial E_1(Pr_i^B(2|10^3,4))}{\partial \lambda} &= \frac{63e^{9\lambda}}{320(1+e^{9\lambda})^2} - \frac{15029e^\lambda}{384000(1+e^\lambda)^2} \\
\frac{\partial E_1(Pr_i^B(2|10^4,4))}{\partial \lambda} &= \frac{3}{3200} \left(\frac{216e^{9\lambda}}{(2+e^{9\lambda})^2} - \frac{146e^\lambda}{(3+2e^\lambda)^2} \right)
\end{aligned}$$

It is easy to show that all the first order differentiations are strictly positive if and only if $0 \leq \lambda < \lambda^*(P, K)$, where $\lambda^*(P, K) \in (0, +\infty)$. That is, there exists a $\lambda^*(P, K) \in (0, +\infty)$ such that the expected probability to buy at price P given cap 10^K increases with λ when $\lambda \leq \lambda^*(P, K)$ and decreases with λ when $\lambda > \lambda^*(P, K)$.

When $\lambda = 0$, the expected probability to buy at period 2 are equal to 80% when price is 1 and cap is 1, and 95.24%, 94.29%, 62.50%, 50%, and 60% at prices 1, 10, 10^2 , 10^3 , and 10^4 and when price cap is 10^4 respectively, which are exactly the same as the initial probabilities. When $\lambda \rightarrow +\infty$, the expected probability to buy at period 2 are 73.33% when price is 1 and cap is 1, and 95.09%, 87.03%, 55.71%, 49.14% and 58.78% at prices 1, 10, 10^2 , 10^3 , and 10^4 and when price cap is 10^4 respectively.

These probabilities are all lower than the initial probabilities. Therefore, there exists a $\lambda'(P, K) \in (0, +\infty)$ such that the expected probability to buy at period 2 is greater than the initial probability if $\lambda \in (0, \lambda'(P, K))$ and lower than the initial probability if $\lambda \in (\lambda'(P, K), +\infty)$.

- In summary, given the initial probabilities to trader from the experimental data, we find that traders choose not to speculate if the observing price is highest or second highest. For other lower prices, traders have incentive to speculate especially λ is not too large.

■

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